

Vortex sub-lattice melting in a two-component superconductor

E. Smørgrav,¹ J. Smiseth,¹ E. Babaev,^{2,1} and A. Sudbø¹

¹*Department of Physics, Norwegian University of Science and Technology, N-7491 Trondheim, Norway*

²*Laboratory of Atomic and Solid State Physics, Cornell University, Ithaca, NY 14853-2501, USA*

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We consider the vortex matter in a three-dimensional two-component superconductor with individually conserved condensates with different bare phase stiffnesses in a finite magnetic field, such as the projected superconducting state of liquid metallic hydrogen. The ground state is a lattice of *composite*, i.e. co-centered, vortices in both order parameters. We investigate quantitatively two novel phase transitions when temperature is increased at fixed magnetic field. *i)* A “vortex sub-lattice melting” phase transition where vortices in the field with lowest phase stiffness (“light vortices”) loose co-centricity with the vortices with large phase stiffness (“heavy vortices”), thus entering a liquid state. Remarkably, the structure factor of the light vortex sub-lattice vanishes *continuously*. This novel transition, which has no counterpart in one-component superconductors, is shown to be in the 3Dxy universality class. Across this transition, the lattice of heavy vortices *remains intact*. *ii)* A first order melting transition of the lattice of heavy vortices, with the novel feature that these are *interacting with a background liquid of light vortices*. These findings are borne out in large-scale Monte Carlo simulations.

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Theories with multi-component bosonic scalar matter fields minimally coupled to a gauge field are of interest in a variety of condensed matter systems and beyond. This includes superconducting low temperature phases of light atoms^{1,2,3,4,5,6} under extreme enough pressures to produce liquid metallic states, easy-plane quantum antiferromagnets⁷, as well as other multiple-component superconductors^{2,3}. It also has applications in particle physics⁸. The projected liquid metallic states of hydrogen (LMH)¹ may soon be realized in high pressure experiments^{9,10}. Moreover, LMH is abundant in the interiors of the giant planets Jupiter and Saturn, where it is the origin of their magnetospheres¹¹. At low temperatures, compressed liquid hydrogen is particularly interesting since it features prominent quantum fluctuations which lead to the possibility of a new state of matter, a near ground state liquid metal¹. Its superconducting counterpart involves Cooper pairs of electrons and protons¹, whence symmetry precludes Josephson coupling between different condensate species. Resolving what happens to such a system in a magnetic field is now a matter of some urgency, due to new and detailed first principles calculations predicting LMH under extreme pressures of order 400GPa¹⁰. This is not far from experimentally achieved pressures of 320GPa⁹, where hints of a maximum in the melting temperature versus pressure are evident. Magnetic-field experiments may very likely be exclusive probes to provide confirmation of LMH. A first study of the phase diagram of the projected LMH in magnetic fields has been presented, unveiling a phase diagram with rich structure⁵. This raises issues of interest also in the broader domain of physics concerning the order and universality classes of possible phase transitions separating phases of partially broken symmetries in quantum fluids. We report a study of this using a confluence of exact topological arguments and large-scale

Monte-Carlo(MC) simulations.

For general number of components N , the Ginzburg-Landau model is defined by the Lagrangian

$$\mathcal{L} = \sum_{\alpha=1}^N \frac{|\mathbf{D}\Psi_0^{(\alpha)}(\mathbf{r})|^2}{2M^{(\alpha)}} + V(\{\Psi_0^{(\alpha)}(\mathbf{r})\}) + \frac{1}{2}(\nabla \times \mathbf{A}(\mathbf{r}))^2. \quad (1)$$

Here, $\{\Psi_0^{(\alpha)}(\mathbf{r}) \mid \alpha = 1 \dots N\}$ are complex scalar fields, $M^{(\alpha)}$ is the mass of the condensate species α , and $\mathbf{D} = \nabla - ie\mathbf{A}(\mathbf{r})$. In LMH *each individual condensate is conserved*, consequently $V(\{\Psi_0^{(\alpha)}(\mathbf{r})\})$ is only a function of $|\Psi_0^{(\alpha)}(\mathbf{r})|^2$. The model is studied in the phase-only approximation $\Psi_0^{(\alpha)}(\mathbf{r}) = |\Psi_0^{(\alpha)}| \exp[i\theta^{(\alpha)}(\mathbf{r})]$ where $|\Psi_0^{(\alpha)}| = \text{const}$. Then, the V -term is a constant which may be omitted from the action⁵.

For the discussions in this paper, another form of the action is useful. Introducing $|\psi^{(\alpha)}|^2 = |\Psi_0^{(\alpha)}|^2/M^{(\alpha)}$ and $\Psi^2 \equiv \sum_{\alpha=1}^N |\psi^{(\alpha)}|^2$, Eq. (1) may be rewritten⁶ in terms of *one* charged and $N-1$ neutral modes

$$\begin{aligned} \mathcal{L} = & \frac{1}{2\Psi^2} \left(\sum_{\alpha=1}^N |\psi^{(\alpha)}|^2 \nabla \theta^{(\alpha)} - e\Psi^2 \mathbf{A} \right)^2 + \frac{1}{2}(\nabla \times \mathbf{A})^2 \\ & + \frac{1}{4\Psi^2} \sum_{\alpha,\beta=1}^N |\psi^{(\alpha)}|^2 |\psi^{(\beta)}|^2 \left(\nabla(\theta^{(\alpha)} - \theta^{(\beta)}) \right)^2. \quad (2) \end{aligned}$$

While Eq. (1) is convenient for MC simulations, Eq. (2) has advantages for analytical considerations, since the neutral and the charged modes are explicitly identified. Moreover, Eq. (2) is convenient for identifying various states of partially broken symmetry, emerging when an N -component system is subjected to an external magnetic field^{5,6}.

We now focus on the case $N = 2$. In the case of LMH, $\Psi_0^{(1)}$ and $\Psi_0^{(2)}$ will denote protonic and electronic superconducting condensates, respectively, and hence $|\psi^{(1)}|^2 \ll |\psi^{(2)}|^2$. In zero external magnetic field this system features a low-temperature phase-transition in the $3Dxy$ universality class at T_{c1} where superfluidity is lost, followed at higher temperatures by an inverted $3Dxy$ transition at T_{c2} where superconductivity and the Higgs mass of \mathbf{A} (Meissner effect) is lost⁴. Here, we will consider the system in finite magnetic field at temperatures below T_{c2} ⁵.

We define a type- α vortex as a topological defect in $\Psi_0^{(\alpha)}$ associated with a non-trivial phase winding $\Delta\theta^{(\alpha)} = \pm 2\pi$, whereas a *composite vortex* is a topological defect where type-1 and type-2 vortices coincide in space. At

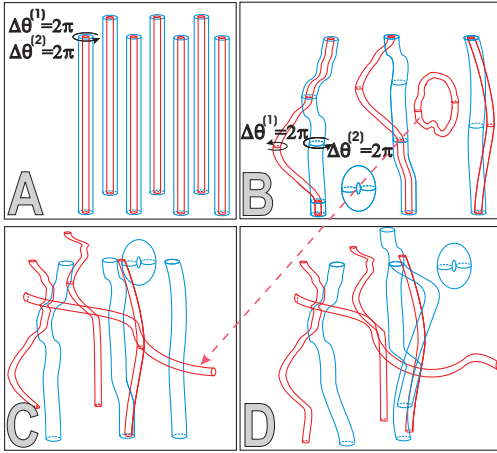


FIG. 1: **Panel A:** A type-II, $N = 2$ superconductor at zero temperature in a magnetic field forms a lattice of composite vortices, i.e. co-centered type-1 (red) and type-2 (blue) vortices. **Panel B:** Low-temperature fluctuations in the composite vortex lattice (VL). Thermal fluctuations generate closed loops of composite vortices and *local splitting* of field-induced composite vortices. This phase features superfluidity, as well as longitudinal superconductivity. **Panel C:** A liquid of type-1 vortices immersed in a background lattice of type-2 vortices. There is a temperature region in low magnetic field when type-1 vortices form a vortex liquid and the corresponding vortex loops are proliferated, while type-2 vortices form a VL. Type-1 and type-2 vortices carry only a fraction of magnetic flux quantum in this state. Superfluidity is lost, longitudinal superconductivity is retained. The arrow from panel B to panel C illustrates type-1 loop-proliferation. **Panel D:** A vortex liquid of type-1 and type-2 vortices. Superfluidity and longitudinal superconductivity is lost, i.e. the normal phase⁵. There is no type-2 loop proliferation going from panel C to panel D, since this is a first-order melting transition of the type-2 VL in the background of a liquid of linetension-less type-1 vortices.

low temperatures the formation of a vortex lattice (VL) of *non-composite vortices* is forbidden because these vortices have a logarithmically divergent energy, whereas *composite* vortices have finite energy^{3,4}. In a type-II 2-component superconductor, therefore, a VL of composite

type-1 and type-2 vortices is formed, illustrated in panel A of Fig. 1. At elevated temperatures, the 2-component system in a magnetic field will exhibit thermal excitations in the form of fractional-flux vortex loops similar to the case of zero magnetic field $\mathbf{B} = \nabla \times \mathbf{A} = 0$ ^{3,4}. Since the field-induced vortices are logarithmically bound states of constituent (elementary) vortices, the thermal fluctuations will induce a *local splitting* of composite vortices in the form of two half-loops connected to a straight line⁵, as shown in panel B of Fig. 1.

Consider now the processes illustrated in panel B of Fig. 1 for the case $|\psi^{(1)}|^2 \ll |\psi^{(2)}|^2$, upon increasing the temperature beyond the low-temperature regime. We may view this process as a type-1 closed vortex loop superposed on a VL of (slightly) fluctuating composite vortices. An important point to notice is that a type- α vortex does not interact with a composite vortex by means of a neutral mode⁶. This follows from a topological argument that two split branches will feature nontrivial-winding in the composite neutral field $\theta^{(1)} - \theta^{(2)}$, while a composite vortex line does not. Hence, the splitting transition may be viewed as a *type-1 vortex loop-proliferation in a neutral superfluid*. This is illustrated in Fig. 2. Thus, we may utilize the well-known results for the crit-

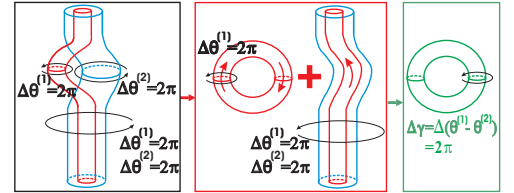


FIG. 2: Detailed illustration of the low-temperature thermal fluctuations in a VL of composite vortices. A local excursion of the vortex component with lowest bare phase stiffness (type-1 vortex) away from the composite VL may be viewed as a type-1 bound vortex loop superposed on the composite VL. The composite vortex line does not interact with a vortex with nontrivial winding in $\Delta\gamma = \Delta(\theta^{(1)} - \theta^{(2)})$ ⁶. A splitting transition of the composite VL, such as illustrated in going from panel B to panel C in Fig. 1, may thus be viewed as a *zero-field vortex-loop proliferation* of type-1 vortices, with a phase transition in the $3Dxy$ universality class^{12,13,14}.

ical properties of the $3Dxy$ model for neutral superfluids described as a vortex-loop proliferation^{12,13,14}. This “vortex sublattice melting” phase transition is therefore in the $3Dxy$ universality class^{12,13,14}, not a first order melting transition. The resulting phase is one where superfluidity is lost and longitudinal superconductivity retained in the component $\Psi_0^{(2)}$ ⁵ illustrated in panel C of Fig. 1.

Apart from the sub-lattice melting transition, thermal fluctuations will produce a melting transition of the type-2 VL at a higher temperature. It is well known that sufficiently strong thermal fluctuations drive a *first-order* melting transition of the Abrikosov lattice¹⁴ in $N = 1$ superconductors. Due to the interplay with the proliferated type-1 vortices, a counterpart to this effect for

the case $N = 2$ when $|\psi^{(1)}|^2 \neq |\psi^{(2)}|^2$ is more complex. The melting temperature $T_M(B)$ of the type-2 Abrikosov lattice is suppressed with increasing magnetic field¹⁴. At low enough magnetic fields, upon heating the system, the 3Dxy type-1 vortex-loop transition at $T_{c1}(B)$ will be encountered before the melting transition of type-2 vortices at $T_M(B)$. Above $T_M(B)$, longitudinal superconductivity is also lost, whence we may infer that the vortex-liquid mixture of liberated type-1 and type-2 vortices is the normal metallic phase⁵, depicted in panel D of Fig. 1.

The above physical picture is borne out in large-scale MC simulations. We consider the model based on Eq. (1) for $N = 2$ on an $L \times L \times L$ lattice with periodic boundary conditions for coupling constants $|\psi^{(1)}|^2 = 0.2$, $|\psi^{(2)}|^2 = 2$, and $e^2 = 1/10$. The ratio $|\psi^{(2)}|^2/|\psi^{(1)}|^2 = 10$ brings out one second order phase transition at $T_{c1}(B)$ in the 3Dxy universality class well below the melting temperature $T_M(B)$ of the VL. In LMH $|\psi^{(2)}|^2/|\psi^{(1)}|^2 \approx 2000$, but the physical picture remains. For real estimates of $T_{c1}(B)$ and $T_M(B)$ in LMH, see Ref. 1. The Metropolis algorithm with local updating is used in combination with Ferrenberg-Swendsen reweighting¹⁵. System sizes as large as $L = 96$ are used. The external magnetic field \mathbf{B} studied is $B^x = B^y = 0$, $B^z = 2\pi/32$, thus there are 32 plaquettes in the (x, y) -plane per flux-quantum. This is imposed by splitting the gauge field into a static part \mathbf{A}_0 and a fluctuating part $\mathbf{A}_{\text{fluct}}$. The former is kept fixed to $(A_0^x, A_0^y(\mathbf{r}), A_0^z) = (0, 2\pi x f, 0)$ where $f = 1/32$ is the magnetic filling fraction, on top of which the latter field is free to fluctuate. Together with periodic boundary conditions on $\mathbf{A}_{\text{fluct}}$, the constraint $\oint_C (\mathbf{A}_0 + \mathbf{A}_{\text{fluct}}) d\mathbf{l} = 2\pi f L^2$, where C is a contour enclosing the system in the (x, y) -plane, is ensured. It is imperative to fluctuate \mathbf{A} , otherwise type-1 and type-2 vortices do not interact^{3,4,6}. To investigate the transition at T_{c1} we have performed finite size scaling (FSS) of the third moment of the action. The simulations are done by using vortices directly, similar to the procedure described in Ref. 4, but with a finite magnetic induction $B^z = 2\pi/32$.

We compute the specific heat and the third moment of the action $S = \beta \int d\mathbf{r} \mathcal{L}$, defined as $M_3 = (\langle S^3 \rangle - \langle S \rangle^3)/L^3$. Here, β is inverse temperature. The peak to peak value of M_3 scales with system size as $L^{(1+\alpha)/\nu}$ and the width between the peaks scales as $L^{-1/\nu}$ ¹⁶. To probe the structural order of the vortex system we compute the planar structure function $S^{(\alpha)}(\mathbf{k}_\perp)$ of the *local vorticity* $\mathbf{n}^{(\alpha)}(\mathbf{r}) = (\nabla \times [\nabla \theta^{(\alpha)} - e \mathbf{A}]) / 2\pi$, given by

$$S^{(\alpha)}(\mathbf{k}_\perp) = \frac{1}{(fL^3)^2} \langle |\sum_{\mathbf{r}} n_z^{(\alpha)}(\mathbf{r}) e^{i\mathbf{k}_\perp \cdot \mathbf{r}_\perp}|^2 \rangle, \quad (3)$$

where \mathbf{r} runs over dual lattice sites and \mathbf{k}_\perp is perpendicular to \mathbf{B} . This function will exhibit sharp peaks for the characteristic Bragg vectors \mathbf{K} of the type- α VL and will feature a ring-structure in its corresponding liquid of type- α vortices. The signature of vortex sub-lattice melting will be a transition from a six-fold symmetric Bragg-peak structure to a ring structure in $S^{(1)}(\mathbf{K})$ while the

peak structure remains intact in $S^{(2)}(\mathbf{K})$. Furthermore, we compute the *vortex co-centrality* N_{co} of type-1 and type-2 vortices, defined as $N_{\text{co}} \equiv N_{\text{co}}^+ - N_{\text{co}}^-$, where

$$N_{\text{co}}^\pm \equiv \frac{\sum_{\mathbf{r}} |n_z^{(2)}(\mathbf{r})| \delta_{n_z^{(1)}(\mathbf{r}), \pm n_z^{(2)}(\mathbf{r})}}{\sum_{\mathbf{r}} |n_z^{(2)}(\mathbf{r})|}, \quad (4)$$

where $\delta_{i,j}$ is the Kronecker-delta. The reason for considering N_{co} is that we then eliminate the effect of random overlap of vortices in the high-temperature phase $T > T_{c1}$ due to vortex-loop proliferation, and focus on the *compositeness* of field-induced vortices. The quantity N_{co} is the fraction of type-2 vortex segments that are co-centered with type-1 vortices, providing a measure of the extent to which vortices of type-1 and type-2 form a *composite* vortex system. Hence, it probes the splitting processes visualized in panel B of Fig. 1 and in Fig. 2. The results are shown in Fig. 3.

The specific heat has a pronounced peak at T_{c1} associated with the 3Dxy transition, and a broader less pronounced peak which is the finite field remnant of the zero-field inverted 3Dxy transition¹³. Scaling of M_3 at T_{c1} shown in the inset c in Fig. 3 yields the critical exponents $\alpha = -0.02 \pm 0.05$ and $\nu = 0.67 \pm 0.03$ in agreement with the 3Dxy universality class. A novel result, not encountered in one-component superconductors, is that the structure function $S^{(1)}(\mathbf{K})$ vanishes *continuously* as the temperature approaches T_{c1} from below, which is precisely the hallmark of the decomposition transition that separates the two types of vortex states depicted in panels B and C in Fig. 1. A related intriguing feature is the *vanishing* in the co-centrality N_{co} at T_{c1} as a function of temperature, which we will discuss in detail below. The first-order melting transition is seen to take place at T_M , where $S^{(2)}(\mathbf{K})$ is seen to vanish discontinuously. This is the temperature at which the translational invariance is restored through melting of the type-2 VL. The resulting translationally invariant high-temperature phase is depicted in panel D of Fig. 1. In the temperature interval $T < T_{c1}$ the system features superconductivity and superfluidity simultaneously⁵, since there is long-range order both in the charged and the neutral vortex modes. In the temperature interval $T_{c1} < T < T_M$ long-range order in the neutral mode is destroyed by loop-proliferation of type-1 vortices, hence superfluidity is lost⁵. However, longitudinal one-component superconductivity is retained along the direction of the external magnetic field. For $T > T_M$ superconductivity is also lost, hence this is the normal metallic state, which is a two-component vortex liquid.

Next, we discuss these results in more detail. The most unusual and surprising feature is the continuous variation of $S^{(1)}(\mathbf{K})$ with temperature, even at T_{c1} where it vanishes. The explanation for this is the proliferation of type-1 vortices (which destroys the neutral superfluid mode) in the background of a composite VL, which the type-1 vortices essentially do not see, cf. Fig. 2. As far as the composite neutral Bose field $\theta^{(1)} - \theta^{(2)}$ is con-

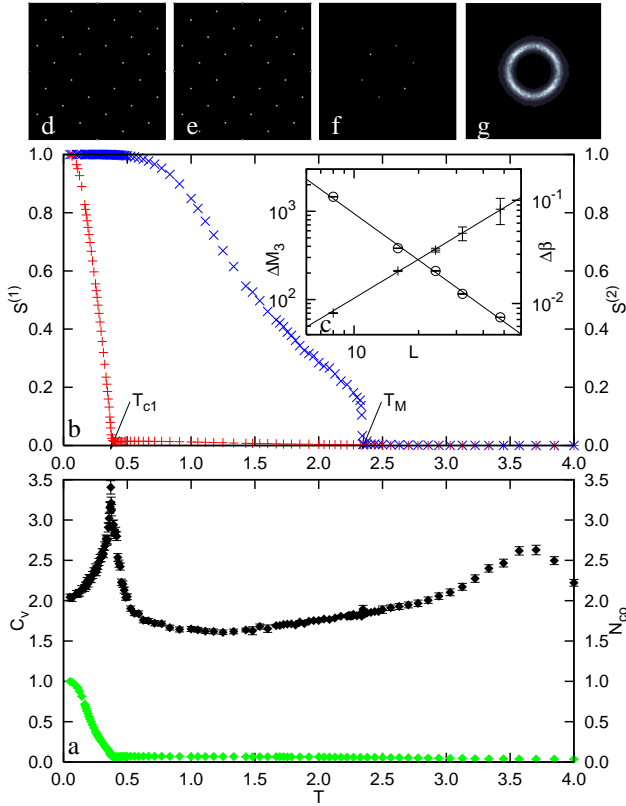


FIG. 3: MC results for $N = 2$ $|\psi^{(1)}|^2 = 0.2$, $|\psi^{(2)}|^2 = 2$, and $e = 1/\sqrt{10}$ are presented. Panel a shows specific heat C_v in black and co-centricity $N_{co} = N_{co}^+ - N_{co}^-$ in green. Note how the specific heat anomaly at $T_{c1} = 0.37$, associated with the proliferation of type-1 vortices, matches the point at which N_{co} drops to zero, illustrating that type-1 vortices tear themselves off type-2 vortices. The remnant of the zero-field anomaly in the specific heat can be seen as a hump at $T \sim 3.6$. Panel b shows the structure factors $S^{(1)}(\mathbf{K})$ in red and $S^{(2)}(\mathbf{K})$ in blue for the particular Bragg vector $\mathbf{K} = (\pi/4, -\pi/4)$. We see that $S^{(1)}(\mathbf{K})$ vanishes continuously at T_{c1} , while $S^{(2)}(\mathbf{K})$ vanishes discontinuously at $T_M = 2.34$. Panel c shows the FSS plots of the M3 from which the exponents $\alpha = -0.02 \pm 0.05$ and $\nu = 0.67 \pm 0.03$ is extracted, showing that the sub-lattice melting is a critical phenomenon in the 3Dxy universality class. Panels d, e, f, and g are gray shade plots of the structure factor $S^{(2)}(\mathbf{k}_\perp)$ for the temperatures $T_d = 0.35$, $T_e = 0.4$, $T_f = 1.66$, and $T_g = 2.85$, respectively. At T_d and T_e on both sides of the specific heat anomaly, the vortex lattice remains intact. The six fold Bragg symmetry is intact at T_f , but is lost at T_M to give a ring pattern at T_g , the hallmark of a vortex liquid.

cerned, it is precisely as if the composite VL were not present at all. Hence, $S^{(1)}(\mathbf{K})$ vanishes for a completely different reason than $S^{(2)}(\mathbf{K})$, namely due to *critical fluctuations*, i.e. *vortex-loop proliferation* in the condensate component with lowest bare stiffness. Such a phase tran-

sition does not completely restore broken translational invariance associated with a VL, since for the type-2 vortices *quite remarkably, the VL order survives the decomposition transition*, due to interaction between heavy vortices mediated by charged modes. The vanishing of N_{co} is particularly interesting, and finds a natural explanation within the framework of the above discussion. That is, for $T \ll T_{c1}$, we have $N_{co} \approx 1$, so the vortex system consists practically exclusively of composite vortices. As the temperature increases, thermal fluctuations induce excursions such as those illustrated in panel B of Fig. 1 and in Fig. 2, which reduces N_{co}^+ from its low-temperature value, reaching a *minimum* at T_{c1} and then *increase* for $T > T_{c1}$. Conversely, N_{co}^- remains essentially zero until T_{c1} , thereafter increasing monotonically. For temperatures above, but close to T_{c1} , fluctuations in vortices originating in $\Delta\theta^{(2)}$ are still small, so the variations in $N_{co} = N_{co}^+ - N_{co}^-$ reflect thermal fluctuations in vortices originating in $\Delta\theta^{(1)}$. The increase of N_{co}^\pm means that type-1 vortex loops are thermally generated, and thus tend to *randomly* overlap more with the moderately fluctuating type-2 vortices. At their first order melting transition, type-2 vortices fluctuate only slightly. *Thus, the vanishing of N_{co} above T_{c1} reflects the increase in the density of thermally generated type-1 vortex loops in the background of a slightly fluctuating type-2 VL*, cf. panel C in Fig. 1.

In summary, we have investigated the orders and universality classes of thermally driven phase transitions in a two-component vortex system in a magnetic field. We find two phase transitions for the fields we consider in this paper (low-field regime). *i)* A 3Dxy phase-transition associated with melting of the VL originating in the phases with lowest stiffness, which may be viewed as vortex-loop proliferation taking place in the background of a composite VL. The structure function associated with the type-1 vortices vanishes *continuously* at the transition. This phase transition has no counterpart in a one-component superconductor. *ii)* A first-order VL melting associated with restoration of translational invariance of the type-2 vortex system. The corresponding structure function vanishes *discontinuously* at the transition, which however takes place in the background of a *liquid of linetensionless type-1 vortices*. This also sets the type-2 VL melting apart from the corresponding phenomenon in one-component superconductors.

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¹ J. Jaffe and N. W. Ashcroft, Phys. Rev. B **23**, 6176 (1981); *ibid* **27**, 5852 (1983); K. Mouloupolos and N. W. Ashcroft,

Phys. Rev. Lett. **66**, 2915 (1991); Phys. Rev. B **59** 12309

- (1999); J. Oliva and N. W. Ashcroft, Phys. Rev. B **30**, 1326 (1984); N. W. Ashcroft, J. Phys. A **129**, 12, (2000); Phys. Rev. Lett. **92**, 187002 (2004).
- ² E. Babaev, L. Faddeev, and A. Niemi, Phys. Rev. B., **65**, 100512 (2002).
 - ³ E. Babaev, Phys. Rev. Lett. **89**, 067001 (2002).
 - ⁴ J. Smiseth, E. Smørgrav, and A. Sudbø, Phys. Rev. Lett. **93**, 077002 (2004).
 - ⁵ E. Babaev, A. Sudbø, and N. W. Ashcroft, Nature **431** 666 (2004).
 - ⁶ J. Smiseth, E. Smørgrav, E. Babaev, and A. Sudbø, to be submitted to Phys. Rev. B.
 - ⁷ T. Senthil, L. Balents, S. Sachdev, A. Vishwanath, and M. P. A. Fisher, Science **303**, 1490 (2004); O. Motrunich and A. Vishwanath, Phys. Rev. B **70**, 075104 (2004); S. Sachdev, in *Lecture Notes in Physics*, U. Schollwöck, J. Richter, D. J. J. Farnell, and R. A. Bishop eds. Springer, Berlin (2004).
 - ⁸ A. J. Niemi, K. Palo and S. Virtanen, Phys. Rev. D **61**, 085020 (2000).
 - ⁹ F. Datchi, P. Loubeyre, and R. le Toullec, Phys. Rev. B **61**, 6535 (2000).
 - ¹⁰ S. A. Bonev, E. Schwegler, T. Ogitsu, and G. Galli, Nature **431**, 669 (2004).
 - ¹¹ T. Guillot, Physics Today **57**, 63 (2004).
 - ¹² Z. Tesanovic, Phys. Rev. B **59**, 6449 (1999).
 - ¹³ A. K. Nguyen and A. Sudbø, Phys. Rev. B **60**, 15307 (1999).
 - ¹⁴ K. Fossheim and A. Sudbø, *Superconductivity: Physics and Applications*, John Wiley & Sons, London (2004).
 - ¹⁵ A. M. Ferrenberg and R. H. Swendsen, Phys. Rev. Lett. **61**, 2635 (1988); *ibid* **63**, 1195 (1989).
 - ¹⁶ A. Sudbø, E. Smørgrav, J. Smiseth, F.S. Nogueira, and J. Hove, Phys. Rev. Lett. **89**, 226403 (2002); J. Smiseth, E. Smørgrav, F.S. Nogueira, J. Hove, and A. Sudbø, Phys. Rev. B, **67**, 205104 (2003).